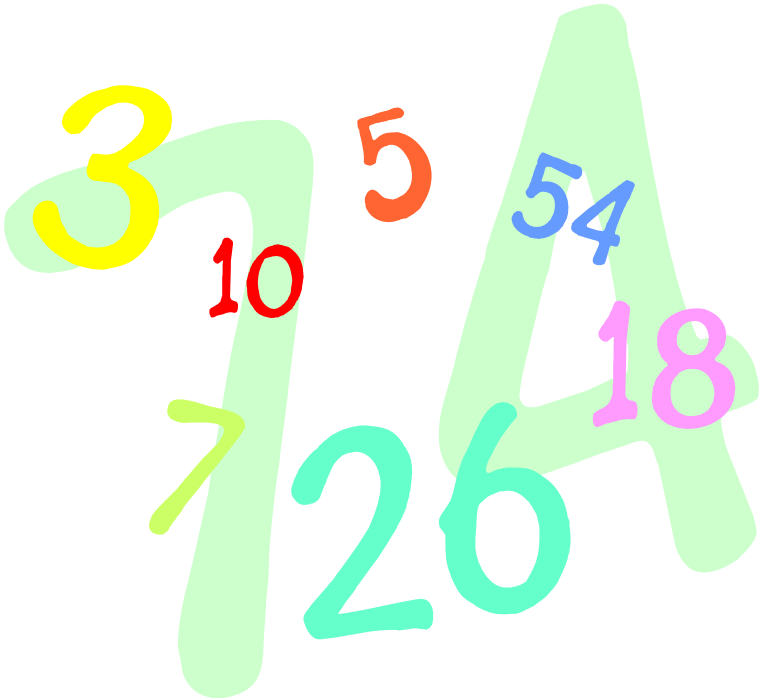
Calculation Policy

Reviewed in January 2020

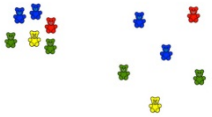


Introduction:

Children are introduced to the processes of calculation through practical, oral and mental activities. As they begin to understand the underlying ideas, they develop ways of recording to support their thinking and calculation methods, so that they develop both **conceptual understanding** and **automaticity** in the fundamentals of mathematics. Whilst interpreting signs and symbols involved with calculation, orally in the first instance, children use models and images to support their mental and written methods of calculation. As children's mental methods are strengthened and refined they begin to work more efficiently, which will support them with using succinct written calculation strategies as they are developed.

<u>From Early Years to Year 1:</u>

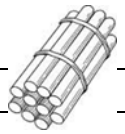
There are fundamental skills that are important for children to develop at an early age. They use this understanding as building blocks for future learning in maths, including calculation. A selection of the skills include:

- Ordinality – 'the ordering of numbers in relation to one another' – e.g. (1, 2, 3, 4, 5…)
- Cardinality – 'understanding the value of different numbers' – e.g. (7 =  17 =  +  14 = 
- Equality – 'seven is the same total as four add three' – e.g.  = 
- Subitising – 'instantly recognizing the number of objects in a small group, without counting them' – e.g.  → five
- Conservation of number – 'recognising that a value of objects are the same, even if they are laid out differently' – e.g. 
- One-to-one correspondence – e.g. 

- Counting on and back from any number – e.g. 'five add three more totals eight'  'ten take away three totals seven' 

- Using apparatus and objects to represent and communicate thinking – e.g.

Date: w/c 1/7/19



(10 straws bundled together)

- Maths language – using mathematical words verbally in every-day situations – e.g. ‘climb up to the top’ / ‘climb down to the bottom’

The ability to calculate mentally forms the basis of all methods of calculation and has to be maintained and refined. A good knowledge of numbers or a ‘feel’ for numbers is the product of structured practice through progression in relevant practical maths experiences and visual representations.

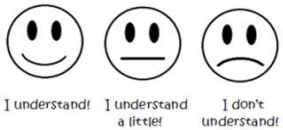
By the end of Year 6, children will be equipped with efficient mental and written calculation methods, which they use with fluency. Decisions about when to progress should always be based on the security of pupils’ understanding and their readiness to progress to the next stage. At whatever stage in their learning, and whatever method is being used, children’s strategies must still be underpinned by a secure understanding and knowledge of number facts that can be recalled fluently.

The overall aims are that when children leave primary school they:

- Are able to recall number facts with fluency, having developed conceptual understanding through being able to visualise key ideas – such as those related to place value - through experience with practical equipment and visual representations;
- Make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads;
- Have an efficient, reliable, written method of calculation for each number operation that they can apply with confidence when undertaking calculations that they cannot carry out mentally;
- Are able to make connections between all four number operations, understanding how they relate to one another, as well as how the rules and laws of arithmetic can be applied.

Children will learn maths through a CPA approach (concrete, pictorial and abstract) and all children will learn to develop ‘automaticity’ when faced with a problem through the teaching of conceptual understanding and the use of mathematical reasoning.

Teachers will use baseline tests to identify children’s gaps and plan effectively for learning. Children who require pre-teaching will have 2 Minute Maths interventions. Teachers will have fluid guided groups in their classrooms and use assembly times and afternoons for over-learning. Teachers will then test after learning has happened to assess progress. Teachers will use the agreed learning objective grids to record AfL Teachers will timetable daily 15 slots for Moonwalk Maths (reviews) and Number Ninja’s (arithmetic practice).

Unit Learning Objectives	I think…	My Teacher thinks…		
	My face is…	Try It! 	Use It! 	Deepen It! 
I can count up and down in hundredths and recognise that one hundredth is one whole divided by 100.		P	G 1:8 (HG)	
I can divide a one or two-digit number by 100 and explain the value of the digits.		P	G 1:8 (HG)	
I can count up and down in hundredths and recognize that one hundredth is also one tenth divided by 10.		I	P	
I can divide a one or two-digit number by 10 and explain the value of the digits.		I	I	P

- AfL- before and after smileys (children's attitudes to their learning)
- Objectives- from Maths Nav/NC and not necessarily day-by-day, but progressing in complexity of concepts
- The three columns (the choice of images is unimportant) represent:

Try It!

This is a whole class activity and is the 'practice' (or fluency) part - a ten minute teaching input and a five minute 'trying' activity for the children in which the children attempt NO MORE than 6 questions. This should be of the expected yearly standard.

If a child requires support or guided teaching during this activity then this will be recorded as **G** in the grid.

If a child requires the support of a partner for this work then they record **P** in the grid.

If a child does this independently they would record an **I** in the grid.

These codes apply to all parts of the grid.

Use it!

This is an independent activity and is the reasoning and problem solving part AT THE EXPECTED STANDARD.

Children requiring guided work will get a **G** in this box; children requiring partner support will get a **P** in this box and children working independently will get an **I** in this box.

Deepen It!

This is also an independent activity involving reasoning and problem solving at 'greater depth'. Children move on to this after successfully completing the **Use It!** activity. Answers can be prepared for children to self-assess if they are able to do so.

'Marking on the Move' (MM) is a good way to monitor the success of the children doing the independent/partner activities before returning to the guided group.

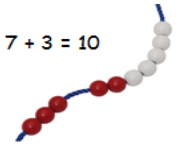
The lesson would still involve an input, guided and independent/pair work, followed by the guided group continuing independently or in pairs with the teacher 'marking' the work of the other children, tackling misconceptions or moving the children on, before returning to a guided group.

Addition:

Mental Calculation Strategies for Addition and Subtraction

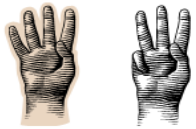
Number Bonds

7 + 3 = 10



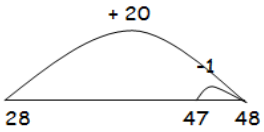
Doubles

Near Doubles



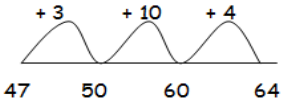
Adjusting

28 + 19 = 47



Bridging

47 + 17 = 64



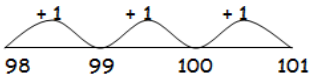
Partitioning

44 + 34 = 78

70 + 8 = 78

Finding the Difference

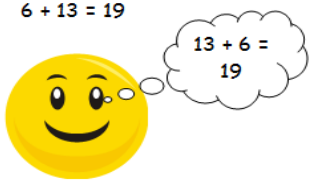
101 - 98 = 3

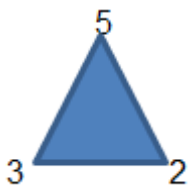
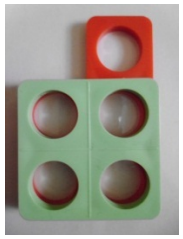
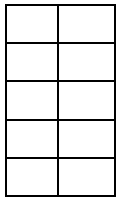
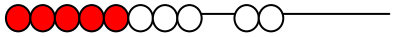
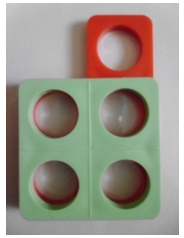


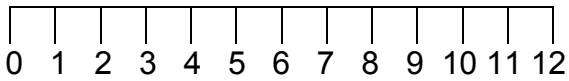
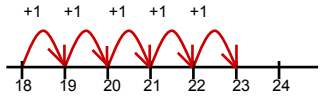
Reordering

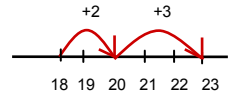
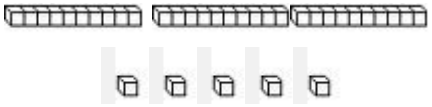
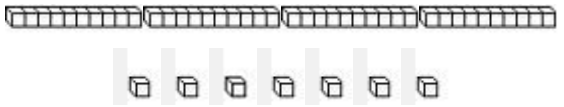
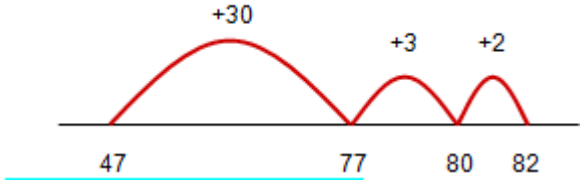
e.g. put big number in head when counting on

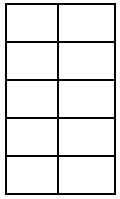
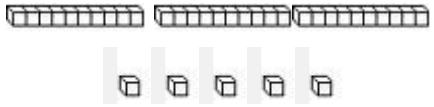
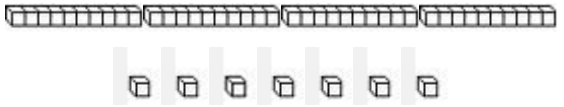
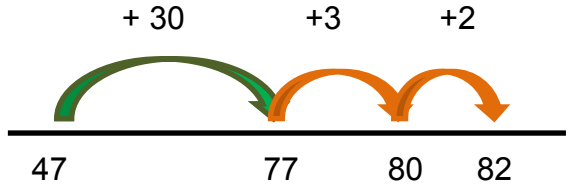
6 + 13 = 19



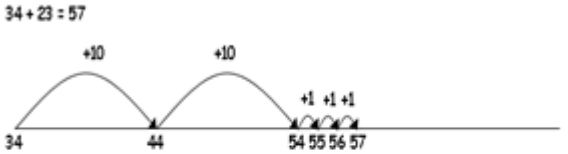
	Counting	Mental maths strategies	Rapid recall	Written calculation and appropriate models and images to support conceptual understanding	
Stage 1:	Count in ones to and across 100 forwards and backwards starting from 0, 1 and other numbers. Count in multiples of two, five and ten.	Pupils use apparatus to explore addition as the inverse of subtraction. .   4 add 1 is 5 5 subtract 4 leaves 1	Rapid recall of all pairs of numbers totalling numbers up to 20. Use structured apparatus – i.e. Numicon, tens frames, abaci, etc. 	**Combining two groups:** • Children develop a mental picture of the number system for use with calculation. A range of key models and images support this, alongside practical equipment. • Teachers model use of number tracks to count on or line up counters/objects along the number track. This is a precursor to use of a fully numbered number-line.	  3 + 2 = 5  ‘eight add two more makes ten’  ‘one more than four is five’

Stage 2:	Continue practicing above skills. Count in steps of 2, 3 and 5 forwards and backwards to and from zero. Count in tens from any	Reorder numbers when adding, i.e. start with largest number, find bonds, etc. Add doubles and derive near doubles. Round numbers to	Recall addition facts for all numbers to 20.	**Counting on from any number:** • Children begin to use numbered lines to support their own calculations, initially counting in ones before beginning to work more efficiently. **Counting on from the largest number:** • Children reorder calculations to	Number line with all numbers labelled  18 + 5 

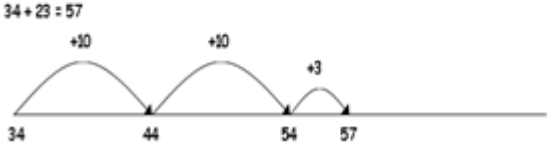
	number – link to coins in a piggy bank as well as a number square.	the nearest 10.		start with the largest number. **Expanded horizontal addition:** • Add numbers using structured apparatus to support understanding of place value. • Make connections between partitioning both numbers using structured apparatus and partition the second number only using a number line.	…to…   Use of questions such as: 'How might I rearrange these to find the total?' Add…  …and…  By partitioning and recombining 30+ 40 = 70 5 + 7 = 12 70 + 12 = 82 35 + 47 

Stage 3:	Continue practicing above skills. Count from 0 in multiples of 4, 8, 50 and 100. Count on by 10 or 100 from any two digit number. Link to counting stick: counting forwards and backwards flexibly. Count up and down in tenths – linking to visual image.	Partitioning by bridging through 10 and multiples of 10 when adding. Adjusting when adding 11 or 9 to a number. Relating inverse number operations – using structured apparatus to explore and understand that subtraction undoes addition.	Connect pairs totalling ten to pairs of multiples of 10 totalling 100.  Use 10ps in tens frame. Recall pairs of two-digit numbers with a total of 100, i.e. 32 + ? = 100.	**Expanded horizontal addition:** • Add numbers using structured apparatus to support understanding of place value. • Make connections between partitioning both numbers using structured apparatus and partition the second number only using a number line. • Teachers need to model how to count on and partition using the colours to represent place value (Green = tens orange = ones) • Teachers model how numbers can be partitioned into tens and ones, as well as in different ways, e.g. 20 + 5 10 + 15 • As children move towards using a columnar method, links continue to be made with earlier models and images, including the number line.	Add…  …and…  By partitioning and recombining 30+ 40 = 70 5 + 7 = 12 70 + 12 = 82  It is crucial that empty number lines are kept as well as using more formal written calculation methods.

Counting on in tens and ones to solve an addition calculation:



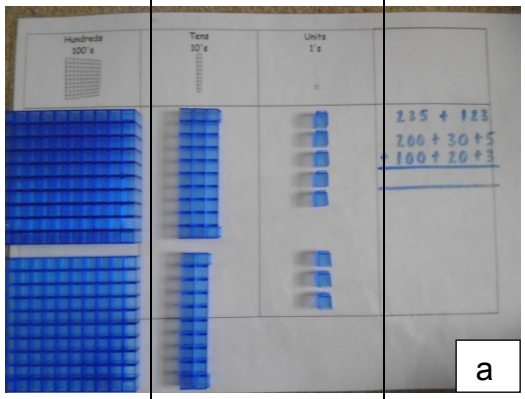
Counting on more efficiently:

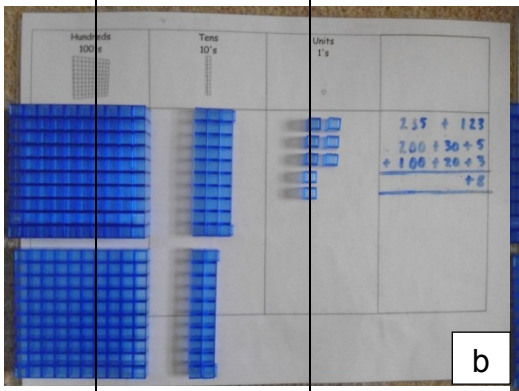


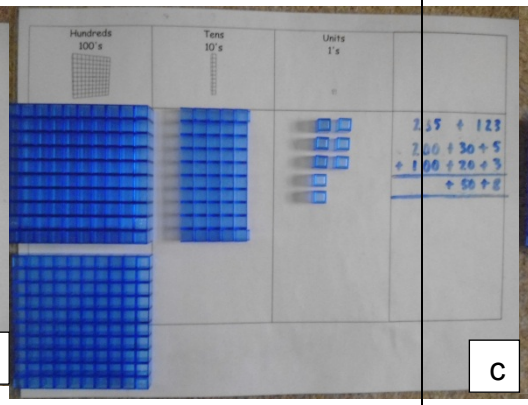
Expanded horizontal method, leading to columnar addition:

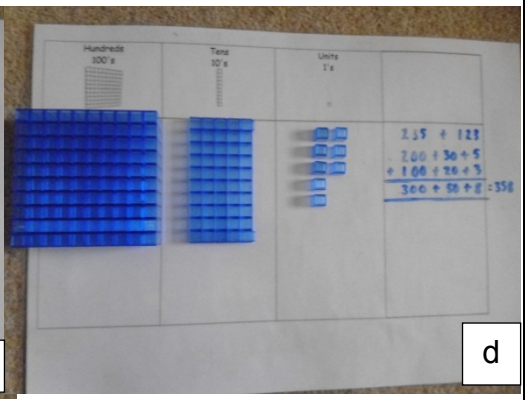
- Written recording should follow teacher modelling around the size of numbers and place value using a variety of concrete materials, e.g. straws, Numicon, Dienes and place-value cards.

Illustration of how to use Dienes equipment to ensure children have an understanding of place value when using columnar addition.









235 + 123

200 + 30 + 5
+ 100 + 20 + 3

235 + 123

200 + 30 + 5
+ 100 + 20 + 3

+ 8

235 + 123

200 + 30 + 5
+ 100 + 20 + 3

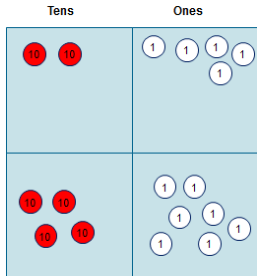
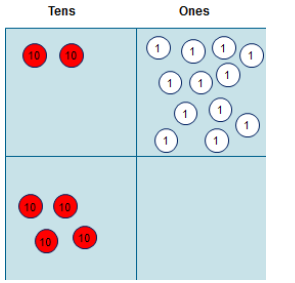
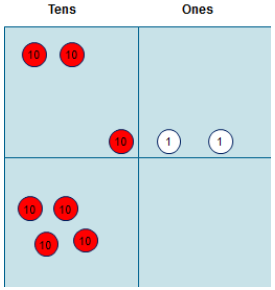
+ 50 + 8

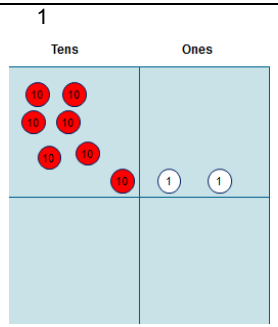
235 + 123

200 + 30 + 5
+ 100 + 20 + 3

+ 300 + 50 + 8 = 358

Stage 4:	Continue practicing previous skills. Count forwards and backwards from 0 in multiples of 6, 7, 9, 25 and 1000 using counting sticks, number lines, number squares, etc. Count up and down in tenths, hundredths and simple fractions using models and images, i.e. Dienes equipment, counting stick, ITPs.	Bridging through 60 for time, i.e. 70 minutes = 1 hour and 10 minutes. Rounding any number to the nearest 10, 100 or 1000. Rounding numbers with one decimal place to nearest whole number. Explore inverse as a way to derive new facts and to check accuracy of answers.	As above. Use known facts and place value to derive new ones, i.e. 'If I know 8 + 3 = 11, I also know 0.8 + 0.3 = 1.1 and 8/100 + 3/100 = 11/100.' Sums and differences of pairs of multiples of 10, 100 or 1000. Addition doubles of numbers to 100. Pairs of fractions totalling 1.	**Expanded columnar addition:** • Teachers model a column method that records and explains partial mental methods. • There remains an emphasis on the language of calculation, e.g. 'Forty plus seventy equals one-hundred and ten.'… 'Seven add six equals thirteen.' …before recombining numbers. Teachers also model the language of: 'Four tens add seven tens total eleven tens or 110.' • Teachers similarly advance to model the addition of two 3-digit numbers with the expectation that as children's knowledge of place value is secured, they become ready to approach a formal compact method. **Columnar addition (formal written method):** • The concept of exchange is introduced through continued use of practical equipment (manipulatives). • Teachers model: 1. "I have two tens and five ones, which need adding to four tens and seven ones."	Adding the ones first: 47 + 76 13 110 123 **Pupils to be encouraged to consider mental strategies first.** Formal columnar (using colours as above to denote place value): 25 +47

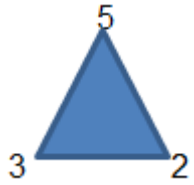
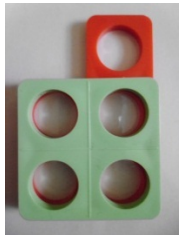
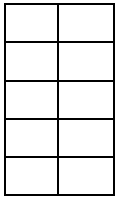
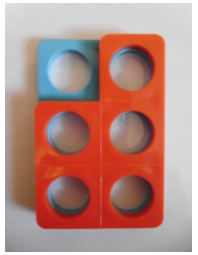
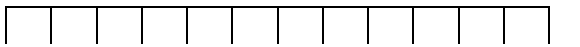
				2. “I add five ones to seven ones, which gives me twelve ones.” 3. “I exchange ten of my twelve ones for a ten counter.” 4. “I add my three tens and four tens to make seven tens.” “Altogether, I have seven tens and two ones.”	 25 +47 2 1  25 +47 2 1  25 +47 72

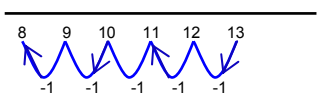
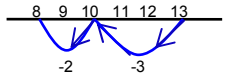
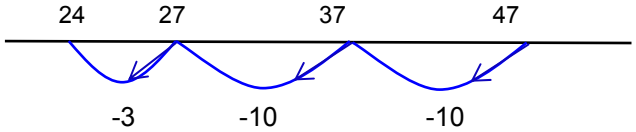
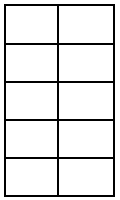
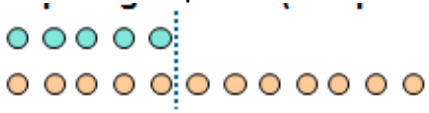
				• Teachers similarly advance to model the addition of two 3-digit numbers, e.g. • Teachers then use this formal compact method to model the addition of two 4-digit numbers.	 587 + 475 1062 1 1

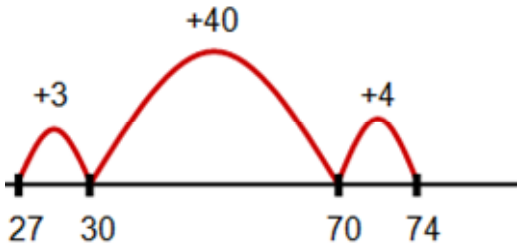
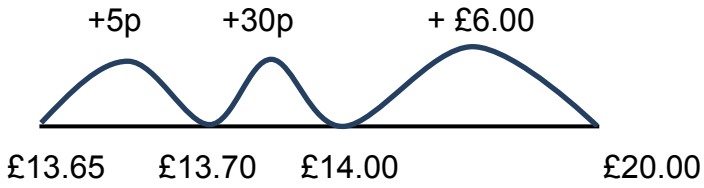
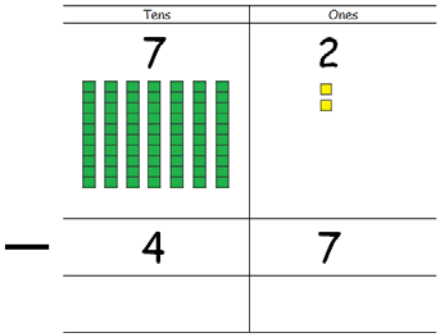
Stage 5:	Count forwards and backwards in steps of powers of 10 for any given number up to one million. Continue to count forwards and backwards in simple fractions. Count forward and backwards in appropriate decimals and percentages.	Use apparatus and knowledge of place value to add decimals, i.e. 3.8 + 2.5 = 5 + 1.3 Reorder increasingly complex calculations, i.e. 1.7 + 2.8 + 0.3 = 1.7 + 0.3 + 2.8 Compensating – i.e. 405 + 399 → add 400 and then subtract 1.	Continue to practice previous stage and make links between known facts and addition pairs for fractions, percentages and decimals Doubles and halves of decimals, i.e. half of 5.6, double 3.4. Sums and differences of decimals, i.e. 6.5 + 2.7	**Columnar addition (formal written method):** • The concept of exchange is continued through use of practical equipment (manipulatives) as modelled above. **Columnar addition (formal written method) with decimal places** • Adding money is a good introduction to adding decimals using 10p, 1p and £1 coins to model.	Model as above Model as above using coins
Stage 6:	Continue to practice previous skills. Count forwards and backwards in simple fractions, decimals and percentages.	Bridging through decimals, i.e. 0.8 + 0.35 = 0.8 + 0.2 + 0.15 using empty number lines. Partitioning using near doubles, i.e. 2.5 + 2.6 = 5 + 0.1	Ensure all children are confident recalling basic facts to 20 and deriving facts using place value. Make links between decimals, fractions and	**Continuation of the above methods**	

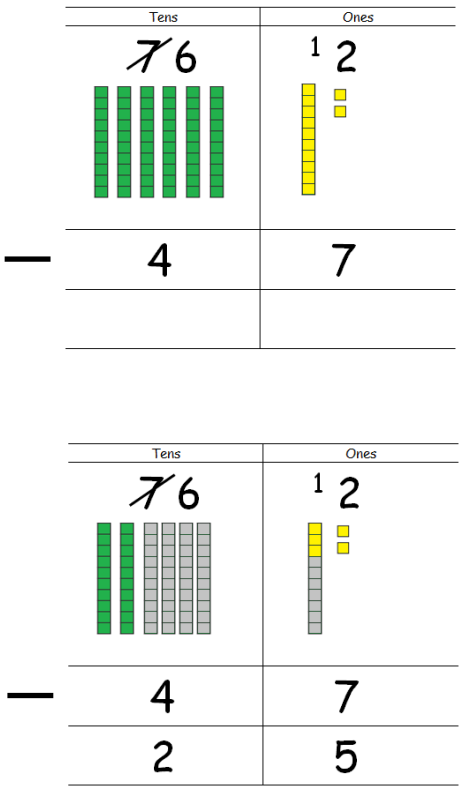
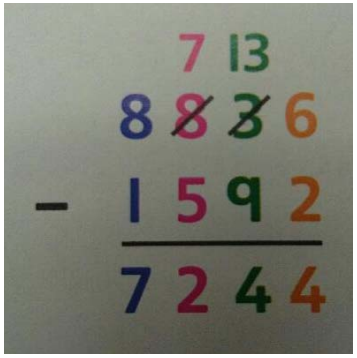
		Reorder decimals, i.e. 4.7 + 5.6 – 0.7 …as… 4.7 – 0.7 + 5.6 = 4 + 5.6.	percentages.		

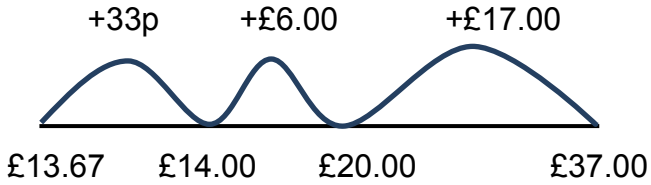
Subtraction:

	Counting	Mental strategies	Rapid Recall	Written calculation and appropriate models and images to support conceptual understanding	
Stage 1:	Count in ones to and across 100, forwards and backwards starting from 0, 1 and other numbers. Count in multiples of two, five and ten.	Pupils use apparatus to explore addition as the inverse of subtraction: .   'four add one is five.' 'five subtract four leaves one'	Rapid recall of subtraction facts for numbers up to 10. Use structured apparatus, i.e. Numicon, tens frames, abaci etc. 	**Subtraction as taking away from a group:** • Children develop a mental picture of the number system for use with calculation. A range of key models and images support this, alongside practical equipment. • Teachers model use of number tracks to count back or remove counters/objects from the number track or set. This is a precursor to use of a fully numbered number-line.	  5 - 2 = 3  'six take away two leaves four'  'one less than six is five'
Stage 2:	Continue practising above skills. Count in steps of 2, 3 and 5,	Bridging through two digit numbers, i.e. 24 – 19 = 19 + 1 + 4 using number lines. Subtracting 11 by subtracting 10 and	Recall subtraction (and addition) facts for all numbers to	**Subtracting by counting back and on:** • Children begin to use numbered lines	Number line with all numbers labelled 

	forwards and backwards to and from zero. Count in tens from any number – link to coins in a piggy bank as well as a number square.	then 1 more. Move to subtracting 9 by subtracting 10 and adding 1 using apparatus.	20.	to support their own calculations, initially counting back in ones before beginning to work more efficiently.	13 – 5 = 8  13 – 5 = 8  47 – 23 = 24 
Stage 3:	Continue practicing above skills. Count from 0 in multiples of 4, 8, 50 and 100. Count on and back by 10 or 100 from any two digit number. Link to counting stick counting forwards and backwards	Partitioning by bridging through 10 and multiples of 10 when subtracting. Continue to practice adjusting when subtracting 11 or 9 from a number. Relating inverse number operations – use structured apparatus to explore and understand that subtraction undoes addition.	Connect subtractions from ten to subtractions from multiples of 10 totalling 100.  Use 10ps in tens frame. Subtract two digit numbers	**Finding the difference:** • Teachers model how to find the difference when two numbers are relatively 'close together.' • Initially children compare two sets before moving on to a number line comparison. • **Pupils are taught to choose whether to**	Comparing two sets: comparison or difference.  Finding the difference on a number line.  Note: Finding the difference is often the most efficient way of solving a subtraction problem, e.g. 61 – 59 2,003 – 1,997 Use empty number lines to find the difference by bridging through multiples of ten.

	flexibly. Count up and down in tenths – linking to visual image.		from 100 i.e. ? = 100 - 78	**count on or back** depending on which is more efficient. E.G 74 – 27 **Finding the difference with larger numbers and money to find change from £10 and £20** E.G Sam has £20. He spends £13.65. How much does he have left? **Using expanded columnar subtraction (3 digits) not stealing**	  89 - 57 = 80 + 9 - 50 + 7 = 30 + 2 = 32
Stage 4:	Continue practicing of previous skills. Count forwards and backwards from 0 in multiples of 6, 7, 9, 25 and 1000 using	Bridging through 60 for time, i.e. 70 minutes = 1 hour and 10 minutes Rounding any number to the nearest 10, 100 or 1000. Rounding numbers with one decimal place to nearest	As above. Use known facts and place value to derive new ones, i.e. 'If I know 11 - 3 = 8, I also know 1.1 - 0.3 = 0.8 and	**Expanded column subtraction with stealing** • The concept of exchange is introduced through continued use of practical equipment	72 – 47 = 

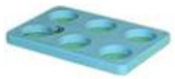
	counting sticks, number lines, number squares, etc. Count up and down in tenths, hundredths and simple fractions using models and images, i.e. Dienes equipment, counting stick, ITPs.	whole number. Explore inverse as a way to derive new facts and to check accuracy of answers.	8/100 - 3/100 = 5/100.' Sums and differences of pairs of multiples of 10, 100 or 1000. Subtraction of fractions totalling 1, i.e. 1 – 0.3 = 0.7	(manipulatives). • Teachers model: 1. "I have seven tens and two ones. I need to subtract four tens and seven ones." 2. "At the moment, I cannot subtract seven ones from two ones, so I need to transfer one ten to become ten ones." 3. "Now I can take away seven ones from twelve ones, so that I have fives ones left." 4. "I can now subtract four tens from six tens, which leaves me with two tens." 5. "I recombine two tens and fives ones to understand that I am left with twenty-five." **Column subtraction with stealing for numbers up to 4 digits**	 

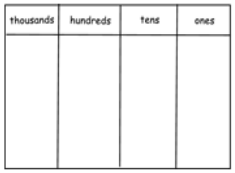
				When moving on to subtracting money there needs to be a discussion about appropriate methods. **Which method is more efficient and why?** When finding change from whole numbers (E.G £37) a number line is more effective because of the amount of stealing/carrying involved in column subtraction. It is important that the numbers are looked at carefully to decide which method to use, considering the margin for error with each.	
Stage 5:	Count forwards and backwards in steps of powers of 10 for any given number up to one million. Continue to count	Use apparatus and knowledge of place value to subtract decimals, i.e. 3.8 - 2.5 = 1.3 Reorder increasingly complex calculations, i.e. 1.7 – 5 – 0.7 = 1.7 – 0.7 – 5.	Continue to practice previous stage and make links between known facts and addition pairs for fractions,	**Column subtraction with stealing for numbers with more than 4 digits.** **Encourage children to estimate their answers and think carefully about the**	Using multi-step problems involving larger numbers and conversion of units.

	forwards and backwards in simple fractions. Count forward and backwards in appropriate decimals and percentages.	Compensating – i.e. 405 - 399 → subtract 400 and then add 1.	percentages and decimals. Doubles and halves of decimals, i.e. half of 5.6, double 3.4. Sums and differences of decimals, i.e. 6.5 + 2.7	**methods they choose to solve the calculations.** **Children should be reflecting on their answers when checking, referring to their estimate and seeing if their answer is reasonable.**	
Stage 6:	Continue to practice previous skills. Count forwards and backwards in simple fractions, decimals and percentages.	Bridging through decimals, i.e. 1.5 – 0.8 = 1.5 – 0.5 then -0.3 using empty number line.	Ensure all children are confident recalling basic facts to 20 and deriving using place value. Make links between decimals, fractions and percentages.	**Continue with column subtraction for numbers with more than 4 digits.**	Using multi-step problems involving larger numbers and conversion of units.

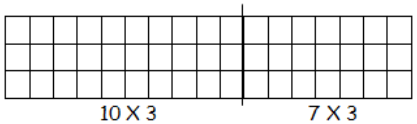
Multiplication:

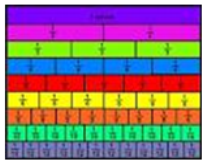
<table>
<tr><td></td><td colspan="5"><u>Mental Calculation Strategies for Multiplication and Division</u>

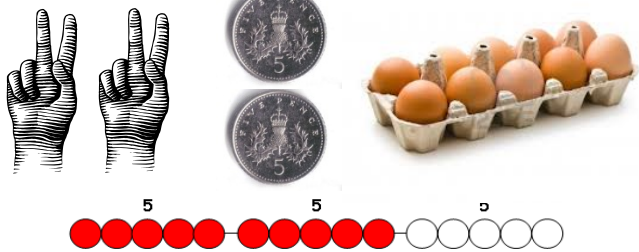
Knowing multiplication and division facts to 12 X 12
  

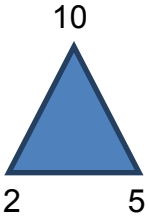
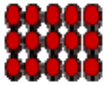
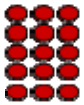
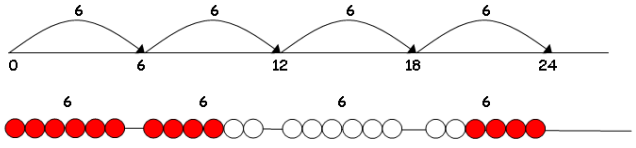
Multiplying and dividing by multiples of 10
  

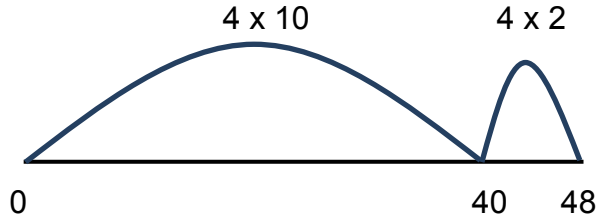
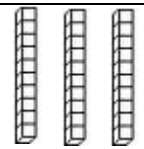
Doubling and halving
 

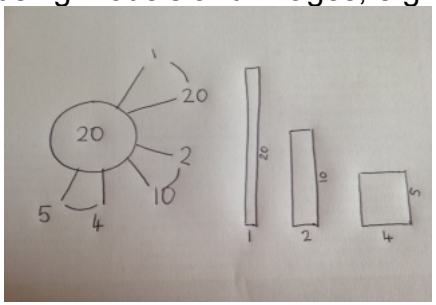
Multiplying and dividing by single-digit numbers and multiplying by two-digit numbers
 
10 X 3 7 X 3
17 X 3

Finding fractions, decimals and percentages
   </td></tr>
<tr><td></td><td>Counting</td><td>Mental strategies</td><td>Rapid recall</td><td colspan="2">Written calculation and appropriate models and images to support conceptual understanding</td></tr>
<tr><td>Stage 1:</td><td>Count forwards and backwards in 2s, 5s and 10s</td><td>Doubling up to six and then ten whilst using related models and images.</td><td>Derive/recall doubles up to five and derive/recall halves up to ten.

Recall odd and even numbers to 10 in reference to</td><td>Developing early conceptual understanding of multiplication:</td><td>Use objects, pictorial representations and arrays to show the concept of multiplication:
 
5 5 5</td></tr>
</table>

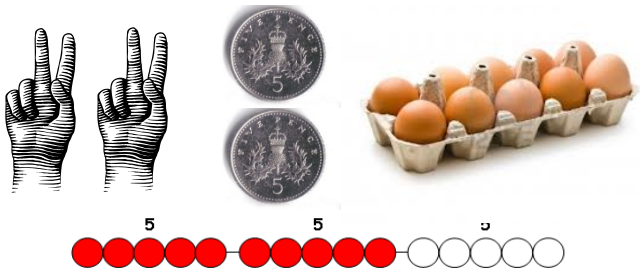
			structured apparatus. 		
Stage 2:	Count forwards and backwards in 2s, 3s, 5s and 10s from zero.	Begin to understand and use inverse number operations:  Stories are used alongside a triad to help children understand links between number operations, e.g. “There are five pencils in two packs, which means that there are ten pencils altogether.”	Derive/recall doubles up to ten and derive/recall halves up to twenty. Recall odd and even numbers to 20 in reference to structured apparatus. Recall & use multiplication facts for the 2X, 5X and 10X-tables.	**Understanding multiplication as repeated addition:** • Investigate multiplication as repeated addition, so that the law of cummutativity is understood. • Whilst arrays are also modelled explicitly at this stage, it is important to note that they will continue to be a key model at later stages, alongside more formal methods of calculation.	Arrays: 5 X 3  and 3 X 5  Number lines: 6 X 4 = 24  So: ‘Six added four times”

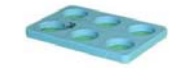
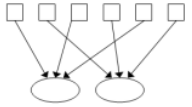
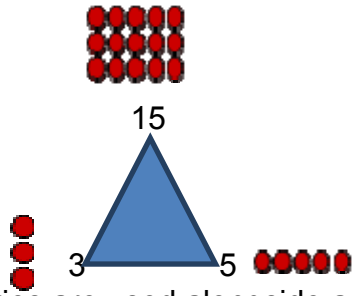
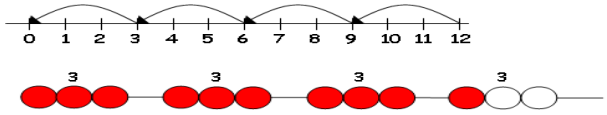
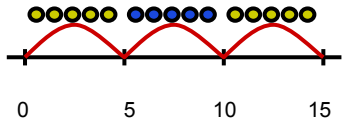
Stage 3:	Counting forwards and backwards in 2s, 3s, 4s, 5s, 8s and 10s from zero. Count up and down in tenths.	Use doubling to make connections between the 2X, 4X and 8X-tables. Understand that multiplication can be undertaken by partitioning numbers, e.g. 12 X 4 = 10 X 4 + 2 X 4 Introduce the structure of scaling: e.g. Find a ribbon that is 4 times as long as the blue ribbon  2cm 8cm	Recall odd and even numbers to 100 in reference to structured apparatus. Recall and use multiplication facts for the 2X, 3X, 4X, 5X, 8X and 10X tables.	**Relate multiplying a 2-digit by 1-digit number using repeated addition and arrays to represent.** **Using expanded short multiplication as a step towards using short multiplication as a formal written method.** Model putting the larger number at the top and then multiplying the digits	Children use an empty number line to jump efficiently: 4 X 12 = 48  3 X 13 = 39 X | 10 | 3 3 |  |  Use models and images to support those that need it. Children expected to record the digits in the boxes (grid method) 7 x 13 = | | 10 | 3 | | 7 | | | Model how to record this using the expanded short multiplication method. 7 X 13 = 13 7 21 (3x7) 70 (10x7) 91

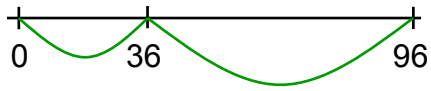
				(using their place value names) starting with the ones column. E.G 3 x 7 then 10 x 7 **NOT** 1 x 7	
Stage 4:	Counting forwards and backwards in 2s, 3s, 4s, 5s, 7s, 8s, 10s, 25s and 1000s from zero. Count up and down in tenths and hundredths.	Derive factor pairs of numbers using models and images, e.g.  Know what happens when a number is multiplied by zero or one. Use reordering to multiply three numbers.	Recall & use multiplication facts for all times-tables up to 12 X 12.	**Relate multiplying a 3/2-digit by 1-digit number by recapping the expanded short multiplication method followed by progressing onto the short multiplication method (without expanding):**	Relate multiplying a 3/2-digit by 1-digit number using the method above. Being able to complete the short multiplication method (without expanding) by the end of stage 4. x 137 6 822 2 4
Stage 5:	Counting forwards and backwards in 2s, 3s, 4s, 5s, 6s, 7s, 8s, 9s, 10s, 25s and 1000s from zero.	Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.	Recall & use multiplication facts for all times-tables up to 12 X 12.	**Relate multiplying a 4/3/2-digit by 1 - digit number using short multiplication.** **Progress onto using long multiplication for multiplying 4/3/2 digit by 2 digit numbers.**	Recap short multiplication method 3 digit by 1 digit then 4 digit by 1 digit. Introduce long multiplication by modelling an expanded example so they can see the whole process but reinforce how this is inefficient and has a big margin for error. Model how to complete the long multiplication method.

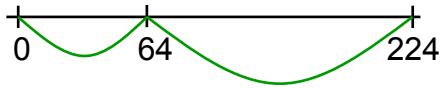
					x 46 12 92 1 460 552 1
Stage 6:	Consolidate all previous counting, including forwards and backwards in fractions.	Perform mental calculations, including with mixed numbers and operations.	Recall & use multiplication facts for all times-tables up to 12 X 12. In addition, use facts confidently to make larger calculations.	**Relate multiplying a 4/3/2-digit by 1/2-digit number using long multiplication:**	Continue with the methods above to secure understanding.

Division:

	Counting	**Mental strategies**	**Rapid recall**	**Written calculation and appropriate models and images to support conceptual understanding**	
Stage 1:	Count forwards and backwards in 2s, 5s and 10s	Doubling up to six and then ten whilst using related models and images.	Derive/recall doubles up to five and derive/recall halves up to ten. Recall odd and even numbers to 10 in	**Developing early conceptual understanding of division as grouping and sharing:**	Use objects, pictorial representations and arrays to show the concept of division as grouping and sharing. 

			reference to structured apparatus.  		"Two children share six pencils between them"  "Six children are asked to get into three equal groups" 
Stage 2:	Count forwards and backwards in 2s, 3s, 5s and 10s from zero.	Begin to understand and use inverse number operations.  Stories are used alongside a triad to help children understand links between number operations, e.g. "15 children are asked to get into three groups and find out that there are five people in each group."	Derive/recall doubles up to ten and derive/recall halves up to twenty. Recall odd and even numbers to 20 in reference to structured apparatus. Recall and use multiplication facts for the 2X, 5X and 10X-tables.	**Understanding division as repeated subtraction:** • Investigate division as repeated subtraction. • Through teacher modelling, children need to know that division is not commutative.	Number lines and arrays: 12 ÷ 3 = 4  15 ÷ 5 = 3 
Stage 3:	Counting forwards and backwards in 2s, 3s,	Use doubling to make connections between the 2X, 4X and 8X-tables. Understand that multiplication	Recall odd and even numbers to 100 in reference to	**Dividing a 2-digit by 1-digit number, representing this efficiently on a number line:**	Children use an empty number line to chunk efficiently. 96 ÷ 6 = 16

	4s, 5s, 8s and 10s from zero.	can be undertaken by partitioning numbers, e.g. 12 X 4 = 10 X 4 + 2 X 4 Introduce the structure of scaling: e.g. Find a ribbon that is 4 times as long as the blue ribbon.  	structured apparatus. Recall & use multiplication facts for the 2X, 3X, 4X, 5X, 8X and 10X tables.	Encouraging children to make links between division and repeated subtraction, using their multiplication facts to 'chunk' larger numbers.	 6 x 6 = 36 10 x 6 = 60 Starting with the dividend (in this case 96) look for easy chunks of the divisor (in this case 6) to take away.
Stage 4:	Counting forwards and backwards in 2s, 3s, 4s, 5s, 7s, 8s, 10s, 25s and 1000s from zero.	Derive factor pairs of numbers using models and images. Know what happens when a number is multiplied by zero or one. Use reordering to multiply three numbers.	Recall & use multiplication facts for all times-tables up to 12 X 12.	**Dividing a 3/2-digit by 1-digit number, using short division.** • At this stage, no remainders are present unless in a practical context.	Children use short division 224 ÷ 8 = 28 $\begin{array}{r} 2\ 8 \\ 8\overline{)\cancel{2}^{2}2^{6}4} \end{array}$
Stage 5:	Counting forwards and backwards in 2s, 3s, 4s, 5s, 6s, 7s, 8s, 9s, 10s, 25s and 1000s from zero.	Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.	Recall & use multiplication facts for all times-tables up to 12 X 12.	**Dividing a 4/3/2-digit by 1-digit number, in relation to short division with remainders.**	Children use short division with remainders 230 ÷ 8 = 28 $\begin{array}{r} 2\ 8\ \text{r}\ 6 \\ 8\overline{)\cancel{2}^{2}3^{7}0} \end{array}$ Remainders should be interpreted in the following ways when short division is used: • as whole numbers (E.G. r6) • as fractions (E.G. 6/8

					or ¾) • through rounding in an appropriate way to the context (E.G 29 buses) where the answer is required to be a decimal then teach as follows: 2 8 . 7 5 8 ⟌ 2 ²3 ⁷0 . ⁶0 ⁴0 Some children might be able to see the link between the fraction remainders and the decimal equivalents. Model this as appropriate.
Stage 6:	Consolidate all previous counting, including forwards and backwards in fractions.	Perform mental calculations, including with mixed numbers and different number operations.	Recall & use multiplication facts for all times-tables up to 12 X 12. In addition, use facts confidently to make larger calculations.	**Dividing a 4/3/2-digit by 2/1-digit number, in relation to long division.** By this stage, there is a statutory requirement that children can use formal written calculation methods, including long and short division.	Make links back to using a number line to chunk efficiently.  8 x 8 = 64 20 x 8 = 160 Long division: 432 ÷ 15 = 28 4/5 2 8 1 5 ⟌ 4 3 2 3 0 0 15×20 1 3 2 1 2 0 15×8 1 2 12/15 = 4/5 Answer: 28 4/5

					or 28.8

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